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Testing the null hypothesis of stationarity against the alternative of a unit root

How sure are we that economic time series have a unit root?*

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We propose a test of the null hypothesis that an observable series is stationary around a deterministic trend. The series is expressed as the sum of deterministic trend, random walk, and stationary error, and the test is the LM test of the hypothesis that the random walk has zero variance. The asymptotic distribution of the statistic is derived under the null and under the alternative that the series is difference-stationary. Finite sample size and power are considered in a Monte Carlo experiment. The test is applied to the Nelson–Plosser data, and for many of these series the hypothesis of trend stationarity cannot be rejected.

1. Introduction

It is a well-established empirical fact that standard unit root tests fail to reject the null hypothesis of a unit root for many economic time series. This was first argued systematically in the influential article of Nelson and Plosser

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(1982), who applied Dickey–Fuller type tests [Dickey (1976), Fuller (1976), Dickey and Fuller (1979)] to 14 annual U.S. time series and failed to reject the hypothesis of a unit root in all but one of the series. These results are not changed by allowing for error autocorrelation using the augmented tests of Said and Dickey (1984) or the test statistics of Phillips (1987) and Phillips and Perron (1988). [We do note, however, that recent work of Choi (1990) that deals with error autocorrelation using feasible GLS methods leads to rather different conclusions.] Similar results are obtained for many other macroeconomic time series. A partial listing of empirical studies yielding these findings can be found in DeJong et al. (1989).

The standard conclusion that is drawn from this empirical evidence is that many or most aggregate economic time series contain a unit root. However, it is important to note that in this empirical work the unit root is the null hypothesis to be tested, and the way in which classical hypothesis testing is carried out ensures that the null hypothesis is accepted unless there is strong evidence against it. Therefore, an alternative explanation for the common failure to reject a unit root is simply that most economic time series are not very informative about whether or not there is a unit root, or equivalently, that standard unit root tests are not very powerful against relevant alternatives. Several more recent studies have argued that this is indeed the case. For example, DeJong et al. (1989) provide evidence that the Dickey–Fuller tests have low power against stable autoregressive alternatives with roots near unity, and Diebold and Rudebusch (1990) show that they also have low power against fractionally integrated alternatives.

Bayesian analysis offers an alternative means of evaluating how informative the data are regarding the presence of a unit root, by providing direct posterior evidence in support of stationarity and nonstationarity. Working from flat priors, DeJong and Whiteman (1991) found only two of the Nelson–Plosser series to have stochastic trends using this approach. Phillips (1991) used objective ignorance priors in extracting posteriors and found support for stochastic trends in five of the series.

These studies suggest that, in trying to decide by classical methods whether economic data are stationary or integrated, it would be useful to perform tests of the null hypothesis of stationarity as well as tests of the null hypothesis of a unit root. This paper provides a straightforward test of the null hypothesis of stationarity against the alternative of a unit root. There have been surprisingly few previous attempts to test the null hypothesis of stationarity. Park and Choi (1988) consider a test statistic which is essentially the F statistic for 'superfluous' deterministic trend variables; this statistic should be close to zero under the stationary null but not under the alternative of a unit root. Rudebusch (1990) considers the Dickey–Fuller test statistics, but estimates both trend-stationary and difference-stationary models and then uses the bootstrap to evaluate the distribution of these statistics under each model. Using the Nelson–Plosser data, he often cannot reject either the trend-stationary model or the difference-stationary model. DeJong et al. (1989) consider the Dickey–Fuller regression

$$y_t = \alpha + \delta t + \rho y_{t-1} + \varepsilon_t, \tag{1}$$

in which the unit root corresponds to $\rho = 1$, but they also test the *stationary* null hypothesis $\rho = 0.85$. For most of the series used by Nelson and Plosser, they can reject neither $\rho = 1$ nor $\rho = 0.85$. Furthermore, this failure to reject both hypotheses is shown to be reasonable in terms of the powers of the tests, which they explore through Monte Carlo experimentation.

These are reasonable first attempts to test stationarity, but they all suffer from the lack of a plausible model in which the null of stationarity is naturally framed as a parametric restriction. Only DeJong et al. test a parametric restriction that implies stationarity, and their choice of $\rho = 0.85$ to represent stationarity (as opposed to $\rho = 0.70$ or 0.95 or whatever) is obviously arbitrary. Clearly stationarity is a composite null hypothesis in models like (1) above.

In this paper we use a parameterization which provides a plausible representation of both stationary and nonstationary variables and which leads naturally to a test of the hypothesis of stationarity. Specifically, we choose a components representation in which the time series under study is written as the sum of a deterministic trend, a random walk, and a stationary error. The null hypothesis of trend stationarity corresponds to the hypothesis that the variance of the random walk equals zero. Under the additional assumptions that the random walk is normal and that the stationary error is normal white noise, the one-sided LM statistic for the trend stationarity hypothesis is the same as the locally best invariant (LBI) test statistic and follows from Nabeya and Tanaka (1988). However, the assumption that the error is white noise is not credible in many applications, since it implies that under the null hypothesis the variable should have iid deviations from trend. We therefore proceed in the spirit of Phillips (1987) and Phillips and Perron (1988) by deriving the asymptotic distribution of the statistics under general conditions on the stationary error, and we propose a modified version of the LM statistic that is valid asymptotically under these general conditions. The asymptotic distribution is nonstandard, involving higher-order Brownian bridges.

When we apply this test to the Nelson–Plosser data, our results depend on the way that the deterministic trend is accommodated. For almost all series we can reject the hypothesis of level stationarity, but for many of the series we cannot reject the hypothesis of trend stationarity. The latter result is in broad agreement with the results of DeJong et al. (1989) and Rudebusch (1990), and with the aforementioned Bayesian analyses of DeJong and Whiteman (1991) and Phillips (1991). It suggests that for many series the existence of a unit root is in doubt, despite the failure of Dickey–Fuller tests (and other unit root tests) to reject the unit root hypothesis.

2. The LM statistic for the stationarity hypothesis

Let y_t, $t = 1, 2, \dots, T$, be the observed series for which we wish to test stationarity. We assume that we can decompose the series into the sum of a deterministic trend, a random walk, and a stationary error:

$$y_t = \xi t + r_t + \varepsilon_t . \tag{2}$$

Here r_t is a random walk:

$$r_t = r_{t-1} + u_t , \tag{3}$$

where the u_t are iid $(0, \sigma_u^2)$. The initial value r_0 is treated as fixed and serves the role of an intercept. The stationarity hypothesis is simply $\sigma_u^2 = 0$. Since ε_t is assumed to be stationary, under the null hypothesis y_t is trend-stationary. We will also consider the special case of the model (2) in which we set $\xi = 0$, in which case under the null hypothesis y_t is stationary around a level (r_0) rather than around a trend.

The statistic we will use is both the one-sided LM statistic and the LBI test statistic for the hypothesis $\sigma_u^2 = 0$, under the stronger assumptions that the u_t are normal and that the ε_t are iid $N(0, \sigma_\varepsilon^2)$. [Because the parameter value specified by the null hypothesis is on the boundary of the parameter space, we are interested in a one-sided LM test rather than a two-sided test; see, e.g., Rogers (1986).] Nyblom and Makelainen (1983) give the LBI statistic for the level-stationary case ($\xi = 0$) of our model. Nyblom (1986) considers a model equivalent to our model and gives the LBI test statistic, but a more convenient expression follows from deriving the statistic as a special case of the statistic developed by Nabeya and Tanaka (1988) to test for random coefficients. [Other relevant references include Tanaka (1983), Franzini and Harvey (1983), and Leybourne and McCabe (1989), and a general discussion can be found in Harvey (1989).] Nabeya and Tanaka consider the regression model

$$y_t = x_t \beta_t + z_t' \gamma + \varepsilon_t , \tag{4}$$

in which the scalar β_t is a normal random walk ($\beta_t = \beta_{t-1} + u_t$, with the u_t iid) and the errors ε_t are iid $N(0, \sigma_\varepsilon^2)$. They test the hypothesis $\sigma_u^2 = 0$, so that they test the null hypothesis of constancy of regression coefficients against the alternative of random walk coefficients. Our model (2) is obviously the special case of their model in which $x_t = 1$ for all t, $z_t = t$, and their

β_t is our r_t. If we set $\xi = 0$ in (2), so as to test the hypothesis of level stationarity, this corresponds to eliminating z_t from their model, in which case we have the simpler model of Nyblom and Makelainen (1986) and Tanaka (1983).

The appendix gives the details of the Nabeya and Tanaka statistic for our model. The end result is very simple. Let e_t, $t = 1, 2, \ldots, T$, be the residuals from the regression of y on an intercept and time trend. Let $\hat{\sigma}_\varepsilon^2$ be the estimate of the error variance from this regression (the sum of squared residuals, divided by T). Define the partial sum process of the residuals:

$$S_t = \sum_{i=1}^{t} e_i, \qquad t = 1, 2, \ldots, T. \tag{5}$$

Then the LM (and LBI) statistic is

$$LM = \sum_{t=1}^{T} S_t^2 / \hat{\sigma}_\varepsilon^2. \tag{6}$$

Furthermore, in the event that we wish to test the null hypothesis of level stationarity instead of trend stationarity, we simply define e_t as the residual from the regression of y on an intercept only (that is, $e_t = y_t - \bar{y}$) instead of as above, and the rest of the construction of the test statistic is unaltered.

The test is an upper tail test. Critical values that are valid asymptotically will be supplied in the next section.

The statistic (6) also may arise in other contexts. Saikkonen and Luukkonen (1990) derive a statistic of the same form as (6) as the locally best unbiased invariant test of the hypothesis $\theta = -1$ in the model $\Delta y_t = v_t + \theta v_{t-1}$, with $\mathrm{E}(y_0)$ unknown and playing the role of intercept in our model, and with the v_t iid normal. [See also Tanaka (1990b).] Note that our model (2) implies that $\Delta y_t = \xi + u_t + \Delta\varepsilon_t$. Define $w_t = u_t + \Delta\varepsilon_t$ as the error in this expression for Δy_t. If u_t and ε_t are iid and mutually independent, w_t has a nonzero one-period autocorrelation, with all other autocorrelations equal to zero, and accordingly it can be expressed as an MA(1) process: $w_t = v_t + \theta v_{t-1}$. Thus our model and the model of Saikkonen and Luukkonen are equivalent to the ARIMA model:

$$y_t = \xi + \beta y_{t-1} + w_t, \qquad w_t = v_t + \theta v_{t-1}, \quad \beta = 1. \tag{7}$$

Let $\lambda = \sigma_u^2/\sigma_\varepsilon^2$. Then the connection between θ and λ is straightforward [see, e.g., Harvey (1989, p. 68)]:

$$\theta = -\left\{(\lambda + 2) - [\lambda(\lambda + 4)]^{1/2}\right\}/2, \qquad \lambda = -(1+\theta)^2/\theta, \tag{8}$$

$$\lambda \geq 0, \quad |\theta| < 1.$$

Thus $\lambda = 0$ corresponds to $\theta = -1$ (stationarity), while $\lambda = \infty$ corresponds to $\theta = 0$ (so y is a pure random walk).

Eq. (7) shows an interesting connection between our tests and the usual Dickey–Fuller tests. The Dickey–Fuller tests test $\beta = 1$ assuming $\theta = 0$; θ is a nuisance parameter. We effectively test $\theta = -1$ assuming $\beta = 0$; now β is the nuisance parameter.

3. Asymptotic theory

In this section we consider the asymptotic distribution of the LM statistic given in (6) above. The LM statistic was derived under the assumption that the errors ε_t are iid $\mathrm{N}(0, \sigma_\varepsilon^2)$. However, in this section we will consider the asymptotic distribution of the statistic under weaker assumptions about the errors. As argued in the Introduction, this is important because the series to which the stationarity test will be applied are typically highly dependent over time, and so the iid error assumption under the null is unrealistic. To allow for quite general forms of temporal dependence we may assume that the ε_t satisfy the (strong mixing) regularity conditions of Phillips and Perron (1988, p. 336) or the linear process conditions of Phillips and Solo (1989, theorems 3.3, 3.14). The Phillips–Perron regularity conditions have been used extensively by subsequent authors, including Leybourne and McCabe (1989). The Phillips–Solo conditions are especially useful because they conveniently allow for all ARMA processes, with either homogeneous or heterogeneous innovations.

Nabeya and Tanaka provide the asymptotic distribution of our test statistics for the case where the ε process is iid, and our results are therefore an extension of theirs. Some of our results are a special case of results in McCabe and Leybourne (1988). [See also Leybourne and McCabe (1989).]

We define the 'long-run variance' as

$$\sigma^2 = \lim_{T \to \infty} T^{-1} \mathrm{E}(S_T^2), \tag{9}$$

which will enter into the asymptotic distribution of the test statistic. A consistent estimator of σ^2, say $s^2(l)$, can be constructed from the residuals e_t, as in Phillips (1987) or Phillips and Perron (1988); specifically, we will use an estimator of the form

$$s^2(l) = T^{-1} \sum_{t=1}^{T} e_t^2 + 2T^{-1} \sum_{s=1}^{l} w(s,l) \sum_{t=s+1}^{T} e_t e_{t-s}. \tag{10}$$

Here $w(s,l)$ is an optional weighting function that corresponds to the choice of a spectral window. We will use the Bartlett window $w(s,l) = 1 - s/(l+1)$

as in Newey and West (1987), which guarantees the nonnegativity of $s^2(l)$. For consistency of $s^2(l)$, it is necessary that the lag truncation parameter $l \to \infty$ as $T \to \infty$. The rate $l = o(T^{1/2})$ will usually be satisfactory under both the null [e.g., Andrews (1991)] and the alternative [see section 4 below].

For the tests of both the level-stationary and trend-stationary hypotheses, the denominator of the LM statistic in (6) is $\hat{\sigma}_\varepsilon^2$, which converges in probability to σ_ε^2. However, when the errors are not iid, the appropriate denominator of the test statistic is an estimate of σ^2 instead of σ_ε^2. To establish this, consider the numerator of the test statistic, normalized by T^{-2}:

$$\eta = T^{-2} \sum S_t^2. \tag{11}$$

We will show that η has an asymptotic distribution equal to σ^2 times a functional of a Brownian bridge, so that division by $s^2(l)$ (or by any other consistent estimate of σ^2) gives an asymptotic distribution free of nuisance parameters.

We consider first the level-stationary case. The model is as in eq. (2) with ξ set to zero, so that the residuals e_t are from a regression of y on intercept only; that is, $e_t = y_t - \bar{y}$. S_t is then the partial sum process of the residuals e, as in eq. (5). Let η_μ be as defined in (11), with the subscript μ indicating that we have extracted a mean but not a trend from y. It is well known that the partial sums of deviations from means of a process satisfying the assumptions of Phillips and Perron (1988) converge to a Brownian bridge, and this implies that

$$\eta_\mu \to \sigma^2 \int_0^1 V(r)^2 \,\mathrm{d}r. \tag{12}$$

Here $V(r)$ is a standard Brownian bridge: $V(r) = W(r) - rW(1)$, where $W(r)$ is a Wiener process (Brownian motion). The symbol $\to$ in (12) signifies weak convergence of the associated probability measures. The limit (12) is a special case of a result obtained previously by McCabe and Leybourne (1988) in the context of tests for random walk regression coefficients.

As noted above, we now divide η_μ by a consistent estimate of σ^2 to get the test statistic that we will actually use. We will indicate this division with a hat (ˆ), so that the test statistic is

$$\hat{\eta}_\mu = \eta_\mu / s^2(l) = T^{-2} \sum S_t^2 / s^2(l). \tag{13}$$

Table 1

Upper tail critical values for $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$.

	η_μ: Upper tail percentiles of the distribution of $\int_0^1 V(r)^2\,\mathrm{d}r$			
Critical level:	0.10	0.05	0.025	0.01
Critical value:	0.347	0.463	0.574	0.739
	η_τ: Upper tail percentiles of the distribution of $\int_0^1 V_2(r)^2\,\mathrm{d}r$			
Critical level:	0.10	0.05	0.025	0.01
Critical value:	0.119	0.146	0.176	0.216

It follows immediately from (12) and from the consistency of $s^2(l)$ that

$$\hat{\eta}_\mu \to \int_0^1 V(r)^2\,\mathrm{d}r. \tag{14}$$

Table 1 gives upper tail critical values of $\int V(r)^2\,\mathrm{d}r$, calculated via a direct simulation, using a sample size of 2000, 50,000 replications, and the random number generator GASDEV/RAN3 of Press, Flannery, Teukolsky, and Vetterling (1986). These critical values agree closely with those given by MacNeill (1978, table 2, p. 431), Nyblom and Makelainen (1983, table 1, p. 859), McCabe and Leybourne (1988, table 3), and Nabeya and Tanaka (1988, table 1, p. 232).

A feasible alternative to simulation is to use numerical integration to invert the characteristic function of the quadratic functional of Brownian motion. This approach is taken by Nabeya and Tanaka (1988) and Tanaka (1990a). It involves some complexity of expression (e.g., the use of Fredholm determinants) but is in principle exact, apart from numerical errors in evaluation of the required integrals.

The analysis of the trend-stationary case is very similar to that of the level-stationary case. The model is now exactly as in eq. (2). Let e_t be the residuals from a regression of y_t on intercept and trend, and let S_t be the partial sum process of the e_t as in (5). Furthermore let η_τ be as defined in (11), where the subscript τ indicates that we have extracted a mean and a trend from y, and serves to distinguish the trend-stationary case from the level-stationary case.

The partial sum process of residuals from a regression of a process satisfying the assumptions of Phillips and Perron (1988) on intercept and trend converges to a so-called second-level Brownian bridge, as given by

MacNeill (1978) or Schmidt and Phillips (1989, app. 3). Thus we have

$$\eta_\tau \to \sigma^2 \int_0^1 V_2(r)^2 \, \mathrm{d}r, \tag{15}$$

where the second-level Brownian bridge $V_2(r)$ is given by

$$V_2(r) = W(r) + (2r - 3r^2)W(1) + (-6r + 6r^2)\int_0^1 W(s)\, \mathrm{d}s. \tag{16}$$

[Our V_2 is MacNeill's B_1 (1978, p. 426).] As previously, we use a hat (^) to indicate that the test statistic has been divided by a consistent estimate of σ^2, and in this notation the test statistic is

$$\hat{\eta}_\tau = \eta_\tau / s^2(l) = T^{-2} \sum S_t^2 / s^2(l). \tag{17}$$

Its asymptotic distribution is given by

$$\hat{\eta}_\tau \to \int_0^1 V_2(r)^2 \, \mathrm{d}r. \tag{18}$$

The upper tail critical values of $\int V_2(r)^2 \, \mathrm{d}r$ are also given in table 1. They agree closely with the critical values given by MacNeill (1974, table 2, p. 431) and Nabeya and Tanaka (1988, table 2, p. 233).

4. Consistency of the test

In this section we consider the asymptotic distributions of the $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ tests under the alternative hypothesis that $\sigma_u^2 \neq 0$. Our interest is in showing that the tests are consistent. This is nontrivial because, under the alternative hypothesis, both the numerator and the denominator of the test statistics diverge. We show that the numerator is $O_p(T^2)$ while the denominator is $O_p(lT)$, so that the test statistic is $O_p(T/l)$. Since $T/l \to \infty$ as $T \to \infty$, the tests are consistent.

We establish this result first for the level-stationary case. We start with the numerator of the statistic, and we first observe that, since the u_t are iid,

$$T^{-1/2} r_{[bT]} = T^{-1/2} \sum_{j=1}^{[bT]} u_j \to \sigma_u W(b), \tag{19}$$

where $b \in [0,1]$ and $[bT]$ is the integer part of bT. Then

$$
\begin{aligned}
T^{-3/2} S_{[aT]} &= T^{-3/2} \sum_{j=1}^{[aT]} (r_j - \bar{r}) + T^{-3/2} \sum_{j=1}^{[aT]} (\varepsilon_j - \bar{\varepsilon}) \\
&= T^{-3/2} \sum_{j=1}^{[aT]} (r_j - \bar{r}) + \mathrm{o}_p(1) \\
&= T^{-1} \sum_{j=1}^{[aT]} T^{-1/2} r_j - ([aT]/T) T^{-1/2} \bar{r} \\
&\to \sigma_u \int_0^a W(b)\,\mathrm{d}b - a\sigma_u \int_0^1 W(b)\,\mathrm{d}b = \sigma_u \int_0^a \underline{W}(s)\,\mathrm{d}s, \quad (20)
\end{aligned}
$$

where $\underline{W}(s)$ is the demeaned Wiener process

$$\underline{W}(s) = W(s) - \int_0^1 W(b)\,\mathrm{d}b. \tag{21}$$

Therefore

$$T^{-4} \sum_{t=1}^{T} S_t^2 = T^{-1} \sum_{t=1}^{T} (T^{-3/2} S_t)^2 \to \sigma_u^2 \int_0^1 \left(\int_0^a \underline{W}(s)\,\mathrm{d}s \right)^2 \mathrm{d}a, \tag{22}$$

so that $T^{-2} \sum S_t^2$ is indeed $\mathrm{O}_p(T^2)$ as claimed in the preceding paragraph.

The argument for the denominator of the test statistic, $s^2(l)$, is more straightforward. From Phillips (1991, unnumbered equation between (A10) and (A11)) we have that

$$(lT)^{-1} s^2(l) \to K\sigma_u^2 \int_0^1 \underline{W}(s)^2\,\mathrm{d}s, \tag{23}$$

provided $T^{-1/2} l \to 0$ as $T \to \infty$. The constant K is defined by

$$K = \int_{-1}^{1} k(s)\,\mathrm{d}s, \tag{24}$$

where $k(s)$ represents the weighting function used in $s(l)$; in our case, $w(s,l) = k(s/l)$ in the notation of eq. (10) above. For the Newey–West estimator, $k(s) = 1 - |s|$ and therefore $K = 1$. Obviously (23) implies that $s^2(l)$ is $\mathrm{O}_p(lT)$.

Since $T^{-2}\Sigma S_t^2$ is $O_p(T^2)$ and $s^2(l)$ is $O_p(lT)$, we deduce that $\hat{\eta}_\mu$ is $O_p(T/l)$. Given that l grows less quickly than T, the test is consistent. However, we have in fact established more than just the order in probability of the test statistic. Under the alternative hypothesis, (22) and (23) imply that

$$(l/T)\,\hat{\eta}_\mu \to \int_0^1 \left(\int_0^a \underline{W}(s)\,\mathrm{d}s \right)^2 \mathrm{d}a \Big/ K \int_0^1 \underline{W}(s)^2\,\mathrm{d}s. \tag{25}$$

Note that this limit is free of nuisance parameters because the scale effect from the variance $\sigma_u^2 \neq 0$ in the numerator and denominator of the limit cancels.

The analysis for the trend-stationary case (i.e., for the statistic $\hat{\eta}_\tau$) is only slightly more complicated. We just need to replace the demeaned Wiener process $\underline{W}(s)$ above with the demeaned and detrended Wiener process $W^*(s)$:

$$W^*(s) = W(s) + (6s-4)\int_0^1 W(r)\,\mathrm{d}r + (-12s+6)\int_0^1 rW(r)\,\mathrm{d}r. \tag{26}$$

This is given by Park and Phillips (1988, eq. (16), p. 474), who prove the equivalent of our (20) above, when S_t is the partial sum process of the residuals of an integrated process on intercept and time trend. The rest of our analysis then follows without further change.

5. Size and power in finite samples

In this section we provide some evidence on the size and power of the $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ tests, in finite samples. Most of this evidence is based on simulations, using the same random number generator as in section 3 and using 20,000 replications.

We first consider the size of the tests in the presence of iid errors. The null hypothesis specifies $\sigma_u^2 = 0$. Furthermore, it is easy to see that the distributions of the statistics under the null do not depend on the parameters r_0, ξ, and σ_ε^2, since the residuals upon which the tests are based do not depend on r_0 or ξ, and the scale factor σ_ε appears in both numerator and denominator and therefore cancels. Thus, the sizes of the tests depend only on sample size (T) and on the number of lags (l) used to calculate $s^2(l)$. Table 2 gives size as a function of T and l. We consider T from 30 to 500, with special emphasis on the relevant range for the Nelson–Plosser data, and we consider three values of l as a function of T: $l0 = 0$, $l4 = \text{integer}[4(T/100)^{1/4}]$, and $l12 =$

Table 2
Size of $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$, 5% level, iid errors.

	$\hat{\eta}_\mu$			$\hat{\eta}_\tau$		
T	*l0*	*l4*	*l12*	*l0*	*l4*	*l12*
30	0.049	0.038	0.004	0.054	0.041	0.248
50	0.050	0.039	0.012	0.052	0.043	0.035
60	0.050	0.040	0.021	0.052	0.043	0.035
70	0.048	0.042	0.025	0.051	0.042	0.033
80	0.049	0.045	0.029	0.049	0.042	0.032
90	0.048	0.043	0.030	0.051	0.045	0.034
100	0.049	0.043	0.029	0.049	0.044	0.033
110	0.050	0.044	0.033	0.051	0.045	0.035
120	0.051	0.045	0.034	0.052	0.046	0.038
200	0.051	0.049	0.041	0.052	0.048	0.040
500	0.050	0.048	0.046	0.052	0.051	0.049

integer$[12(T/100)^{1/4}]$. These choices of l follow Schwert (1989) and other recent simulations.

We can see in table 2 that the tests have approximately correct size except when T is small and l is large. For $l = 0$, the tests have correct size even for $T = 30$, so that the asymptotic validity of the tests holds even for fairly small samples. Using $l = l4$, the tests are slightly less accurate, and the improvement as T increases is slow. For $l = l12$, there are considerable size distortions for $T = 30$ and moderate distortions (too few rejections) even for $T = 100$ or 200, though the tests are quite accurate for $T = 500$. Unsurprisingly, the larger l, the larger is the sample size required for the asymptotic results to be relevant.

We next consider the size of the tests in the presence of autocorrelated errors. In particular, we will consider AR(1) errors, of the form $\varepsilon_t = \rho\varepsilon_{t-1} + \gamma_t$, with the γ_t iid. The AR(1) parameter ρ is a convenient nuisance parameter to consider, since it naturally measures the distance of the null from the alternative. In particular, under the null that $\sigma_u^2 = 0$, y_t becomes a random walk as $\rho \to 1$. As a result, we expect a problem of overrejection for $\rho > 0$, with its severity depending on how close ρ is to unity.

Table 3 presents our simulation results giving the size of the tests for $\rho = 0$, ± 0.2, ± 0.5, and ± 0.8, and for T between 30 and 500. As expected, the tests reject too often for $\rho > 0$ and too seldom for $\rho < 0$. The overrejection problem is very severe for $l = 0$, which is not surprising since the test is not valid even asymptotically in this case. However, the *l4* and *l12* versions of the tests do not improve very rapidly with sample size. The *l4* tests have moderate size distortions for $\rho = 0.5$ and considerable distortions for $\rho = 0.8$, while the *l12* tests are fairly good for $T \geq 30$ and $\rho \leq 0.5$, but not so good for $\rho = 0.8$. Unfortunately, $\rho = 0.8$ is a plausible parameter value since, if we

Table 3

Size of $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$, 5% level, AR(1) errors.

		$\hat{\eta}_\mu$			$\hat{\eta}_\tau$		
ρ	T	*l0*	*l4*	*l12*	*l0*	*l4*	*l12*
0.8	30	0.654	0.301	0.007	0.769	0.317	0.124
	50	0.725	0.264	0.039	0.886	0.319	0.057
	80	0.779	0.300	0.080	0.936	0.401	0.084
	100	0.796	0.250	0.081	0.952	0.337	0.092
	120	0.807	0.256	0.091	0.960	0.354	0.104
	200	0.833	0.271	0.094	0.977	0.396	0.108
	500	0.852	0.239	0.092	0.989	0.361	0.111
0.5	30	0.321	0.114	0.005	0.425	0.129	0.178
	50	0.331	0.098	0.021	0.486	0.113	0.047
	80	0.350	0.108	0.042	0.521	0.124	0.046
	100	0.352	0.090	0.043	0.538	0.107	0.047
	120	0.359	0.092	0.047	0.542	0.114	0.054
	200	0.367	0.099	0.053	0.559	0.121	0.054
	500	0.370	0.090	0.058	0.586	0.110	0.062
0.2	30	0.118	0.055	0.004	0.147	0.062	0.227
	50	0.118	0.053	0.015	0.156	0.059	0.045
	80	0.122	0.060	0.033	0.157	0.060	0.036
	100	0.123	0.054	0.033	0.159	0.057	0.038
	120	0.125	0.057	0.038	0.166	0.064	0.042
	200	0.128	0.061	0.045	0.168	0.065	0.043
	500	0.129	0.059	0.049	0.170	0.065	0.052
− 0.2	30	0.017	0.025	0.003	0.016	0.027	0.268
	50	0.015	0.029	0.011	0.012	0.029	0.039
	80	0.014	0.034	0.024	0.011	0.031	0.028
	100	0.014	0.033	0.026	0.010	0.031	0.029
	120	0.014	0.036	0.029	0.013	0.035	0.034
	200	0.014	0.038	0.037	0.011	0.036	0.036
	500	0.014	0.013	0.042	0.010	0.039	0.042
− 0.5	30	0.002	0.010	0.002	0.001	0.010	0.301
	50	0.001	0.015	0.007	0.001	0.016	0.032
	80	0.001	0.018	0.017	0.001	0.015	0.021
	100	0.001	0.019	0.020	0.000	0.016	0.021
	120	0.001	0.020	0.023	0.000	0.019	0.025
	200	0.001	0.021	0.030	0.000	0.020	0.029
	500	0.001	0.026	0.036	0.000	0.024	0.036
− 0.8	30	0.000	0.002	0.001	0.000	0.001	0.319
	50	0.000	0.007	0.002	0.000	0.013	0.028
	80	0.000	0.008	0.007	0.000	0.008	0.013
	100	0.000	0.007	0.007	0.000	0.002	0.009
	120	0.000	0.003	0.010	0.000	0.002	0.011
	200	0.000	0.008	0.015	0.000	0.002	0.015
	500	0.000	0.010	0.022	0.000	0.008	0.020

Table 4

Power of $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$, 5% level, iid errors.

		$\hat{\eta}_\mu$			$\hat{\eta}_\tau$		
T	λ	$l0$	$l4$	$l12$	$l0$	$l4$	$l12$
30	0.0001	0.050	0.038	0.004	0.053	0.040	0.243
	0.001	0.058	0.046	0.004	0.054	0.042	0.244
	0.01	0.146	0.110	0.009	0.080	0.056	0.240
	0.1	0.514	0.403	0.034	0.287	0.189	0.200
	1	0.806	0.600	0.053	0.725	0.431	0.152
	100	0.883	0.639	0.059	0.875	0.485	0.147
	10000	0.887	0.641	0.059	0.888	0.508	0.141
50	0.0001	0.051	0.041	0.013	0.054	0.041	0.045
	0.001	0.075	0.060	0.020	0.060	0.047	0.048
	0.01	0.287	0.232	0.089	0.129	0.096	0.065
	0.1	0.721	0.566	0.267	0.547	0.357	0.129
	1	0.924	0.683	0.332	0.914	0.579	0.171
	100	0.958	0.703	0.342	0.947	0.608	0.174
	10000	0.959	0.704	0.343	0.974	0.627	0.176
100	0.0001	0.063	0.055	0.038	0.054	0.047	0.036
	0.001	0.168	0.147	0.100	0.084	0.070	0.053
	0.01	0.587	0.508	0.376	0.352	0.278	0.172
	0.1	0.927	0.762	0.551	0.878	0.675	0.357
	1	0.989	0.818	0.579	0.993	0.810	0.411
	100	0.994	0.826	0.584	0.999	0.825	0.417
	10000	0.998	0.827	0.582	0.999	0.820	0.410
200	0.0001	0.097	0.092	0.078	0.065	0.060	0.051
	0.001	0.399	0.372	0.314	0.193	0.174	0.132
	0.01	0.846	0.776	0.626	0.729	0.645	0.448
	0.1	0.990	0.924	0.713	0.990	0.922	0.637
	1	0.999	0.943	0.725	1.00	0.956	0.667
	100	1.00	0.945	0.726	1.00	0.961	0.672
	10000	1.00	0.947	0.725	1.00	0.966	0.675
500	0.0001	0.307	0.295	0.275	0.137	0.132	0.118
	0.001	0.788	0.757	0.682	0.621	0.583	0.503
	0.01	0.997	0.962	0.865	0.983	0.957	0.843
	0.1	1.00	0.989	0.897	1.00	0.996	0.903
	1	1.00	0.992	0.901	1.00	0.998	0.911
	100	1.00	0.992	0.901	1.00	0.998	0.911
	10000	1.00	0.992	0.901	1.00	0.998	0.911

take most series to be stationary, their first-order autocorrelations will often be in this range.

Finally, we consider the power of the tests. Table 4 presents simulation results giving the powers of the tests in the presence of iid errors, as a function of the only two relevant parameters, T and λ, where as before $\lambda = \sigma_u^2/\sigma_\varepsilon^2$. This is done for T between 30 and 500 and λ between 0.0001 and 10,000.

For a given T, power increases with λ (except for the $l12$ version of $\hat{\eta}_\tau$ at $T=30$), as expected. However, as $\lambda \to \infty$ power approaches a limit that is not necessarily unity. Since $\lambda = \infty$ corresponds to $\theta = 0$ in the ARIMA representation (7), this is reasonable. The limiting powers as $\lambda \to \infty$ are well predicted by our asymptotics under the alternative, as given by eq. (25) of section 4. (Recall that the presence of stationary error does not affect the asymptotic distribution of the statistics under the alternative. In that sense these asymptotics correspond to $\sigma_\varepsilon^2 = 0$, or $\lambda = \infty$.) For example, for $\hat{\eta}_\tau$ with $l = 0$ and $T = 30$, the actual (simulation) power of 0.888 (for $\lambda = 10{,}000$) compares to 0.880 predicted by (25). Similarly, for $T = 100$ and $l = 12$, the actual power of 0.410 compares to 0.417 predicted by (25).

Conversely, the power of the $\hat{\eta}_\mu$ test for $l = 0$ and for small λ is well predicted by the asymptotics of Tanaka (1990b), who considers asymptotic behavior as $T \to \infty$, with $\theta = -1 + c/T$ and c fixed. Thus as $T \to \infty$ he has $\theta \to -1$, or $\lambda \to 0$. For example, for $T = 30$ and $\lambda = 0.1$, the actual (simulation) power of 0.514 compares to the predicted power of 0.505, and for $T = 200$ and $\lambda = 0.001$ the actual power of 0.399 compares to predicted power of 0.400.

Power also increases as T increases for fixed λ (again, except for small T and $l = l12$), which is presumably a reflection of the consistency of the tests. The rate at which this happens depends strongly on l. Again, this is as predicted by our asymptotics under the alternative. It should be stressed that the distribution of our tests under the alternative depends on l (i.e., on l/T) *even asymptotically*, so that there is a clear supposition that choosing l larger will cost power, as indeed it does in our simulations. For the $\hat{\eta}_\tau$ test with $T \leq 100$, for example, power with $l = l12$ is never larger than 0.42 no matter how large λ is. With $l = l4$, on the other hand, there is reasonable power for T in the empirically relevant range of 50 to 100 if λ is larger than, say, 0.1. Thus for T in this range there is a clear, if unattractive, trade-off between correct size and power: choosing l large enough to avoid size distortions in the presence of realistic amounts of autocorrelation will make the tests have very little power. Only for $T \geq 200$ do we find appreciable power without the risk of very substantial size distortions.

6. Application to the Nelson–Plosser data

In this section we apply our tests for stationarity to the data analyzed by Nelson and Plosser (1982). These are U.S. annual data covering from 62 to 111 years and ending in 1970. These data have been analyzed subsequently by many others, including Perron (1988) and DeJong et al. (1989). A rough assessment of their findings is as follows. For 12 of the 14 series, we clearly cannot reject the null hypothesis of a unit root. The unit root hypothesis is generally rejected for the unemployment rate series and the industrial

Table 5

$\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ tests for trend stationarity applied to Nelson–Plosser data.

	Lag truncation parameter (l)								
Series	0	1	2	3	4	5	6	7	8
	η_μ: 5% critical value is 0.463								
Real GNP	5.96	3.06	2.08	1.59	1.30	1.11	0.97	0.86	0.78
Nominal GNP	5.81	2.98	2.04	1.56	1.28	1.09	0.95	0.85	0.77
Real per capital GNP	5.54	2.84	1.94	1.50	1.22	1.05	0.92	0.82	0.75
Industrial production	10.79	5.48	3.70	2.81	2.27	1.92	1.66	1.47	1.32
Employment	7.57	3.87	2.63	2.01	1.64	1.39	1.21	1.08	0.98
Unemployment rate	0.31	0.18	0.14	0.11	0.10	0.10	0.09	0.09	0.09
GNP deflator	7.51	3.82	2.59	1.97	1.60	1.35	1.18	1.04	0.94
Consumer prices	7.90	4.02	2.73	2.08	1.69	1.43	1.24	1.10	0.99
Wages	6.72	3.43	2.33	1.78	1.45	1.23	1.07	0.95	0.86
Real wages	6.96	3.55	2.40	1.83	1.48	1.26	1.09	0.97	0.88
Money	8.01	4.08	2.76	2.10	1.70	1.44	1.25	1.11	1.00
Velocity	8.40	4.29	2.90	2.21	1.80	1.52	1.32	1.17	1.05
Interest rate	0.78	0.42	0.30	0.24	0.20	0.17	0.16	0.14	0.13
Stock prices	8.01	4.10	2.79	2.13	1.74	1.48	1.29	1.15	1.04
	$\hat{\eta}_\tau$: 5% critical value is 0.146								
Real GNP	0.630	0.337	0.242	0.198	0.173	0.158	0.148	0.141	0.137
Nominal GNP	0.755	0.392	0.273	0.215	0.181	0.159	0.143	0.132	0.124
Real per capital GNP	0.528	0.283	0.204	0.167	0.147	0.134	0.126	0.121	0.118
Industrial production	0.822	0.446	0.320	0.257	0.220	0.196	0.179	0.166	0.155
Employment	0.526	0.278	0.198	0.158	0.136	0.122	0.112	0.105	0.101
Unemployment rate	0.216	0.124	0.094	0.079	0.071	0.066	0.063	0.061	0.061
GNP deflator	0.492	0.256	0.178	0.140	0.117	0.103	0.093	0.086	0.081
Consumer prices	1.85	0.943	0.641	0.491	0.401	0.342	0.301	0.270	0.246
Wages	0.612	0.317	0.220	0.173	0.145	0.128	0.115	0.107	0.101
Real wages	0.956	0.511	0.365	0.293	0.252	0.226	0.208	0.194	0.184
Money	0.445	0.228	0.158	0.124	0.104	0.092	0.084	0.079	0.075
Velocity	1.78	0.932	0.647	0.504	0.418	0.360	0.319	0.287	0.262
Interest rate	0.845	0.457	0.323	0.255	0.214	0.186	0.166	0.151	0.140
Stock prices	1.23	0.646	0.454	0.359	0.302	0.264	0.237	0.216	0.199

production series. The conventional wisdom is that these results indicate the presence of a unit root in most of the Nelson–Plosser series. We wish to check whether our approach to testing stationarity corroborates this reading of the data.

In table 5 we first present the $\hat{\eta}_\mu$ test statistic for the null hypothesis of stationarity around a level. We consider values of the lag truncation parameter l (used in the estimation of the long-run variance) from zero to eight. The choice of eight as the maximal value of l is based on two considerations. First, for most of the series the value of the long-run variance estimate has settled down reasonably by the time we reach $l = 8$, and so the value of the test statistic has also settled down. Second, based on the simulations of the previous section, $l = 8$ is a compromise between the large size distortions under the null that we would expect for $l = 4$ and the very low power under the alternative that we would expect for $l = 12$. Unfortunately, the values of the test statistics are fairly sensitive to the choice of l, and in fact for every series the value of the test statistic decreases as l increases. This occurs because $s^2(l)$ increases as l increases, and is a reflection of large and persistent positive autocorrelations in the series. Nevertheless, the outcome of the tests is not in very much doubt: for all series except the unemployment rate and the interest rate, we can reject the hypothesis of level stationarity.

The ability to reject the hypothesis of level stationarity is not very surprising in light of the obvious deterministic trends present in these series. We therefore proceed to test the null hypothesis of stationarity around a deterministic linear trend, for which $\hat{\eta}_\tau$ is the appropriate statistic. Once again the test statistics decline monotonically as l increases, and in this case the choice of l is important to the conclusions. If we did not correct for error autocorrelation at all, which corresponds to picking $l = 0$, we would reject the null hypothesis of trend stationarity for every series. As argued above, for temporally dependent series such as the ones under consideration, iid errors are not plausible under the null hypothesis, and failing to allow for autocorrelation is not recommended. Using the results for $l = 8$, we find that we can reject the hypothesis of trend stationarity at the 5% level for five series: industrial production, consumer prices, real wages, velocity, and stock prices. For three other series (real GNP, nominal GNP, and interest rate) we can reject the hypothesis of trend stationarity at the 10% level. We cannot reject the null hypothesis of trend stationarity at usual critical levels for six series: real per capita GNP, employment, unemployment rate, GNP deflator, wages, and money. These empirical results seem to be very much in accord with the Bayesian posterior analysis in Phillips (1991).

Combining the results of our tests of the trend stationarity hypothesis with the results of the Dickey–Fuller tests, the following picture emerges. The unemployment series appears to be stationary, since we can reject the unit root hypothesis and cannot reject the trend stationarity hypothesis. Four series (consumer prices, real wages, velocity, and stock prices) appear to have unit roots, since we can reject the trend stationarity hypothesis and cannot reject the unit root hypothesis. Three more series (real GNP, nominal GNP, and the interest rate) probably have unit roots, though the evidence against the trend stationarity hypothesis is only marginally significant. For six series (real per capita GNP, employment, unemployment rate, GNP deflator, wages, and money) we cannot reject either the unit root hypothesis or the trend stationarity hypothesis, and the appropriate conclusion is that the data are not sufficiently informative to distinguish between these hypotheses. Finally, for the industrial production series, there is evidence against both hypotheses, and thus it is not clear what to conclude. Presumably other alternatives, such as explosive roots, fractional integration, or stationarity around a nonlinear trend, could be considered.

7. Concluding remarks

We have presented statistical tests of the hypothesis of stationarity, either around a level or around a linear trend. These tests could be extended to allow for nonlinear trends, along the same lines as in Schmidt and Phillips (1989, sect. 5). The tests are intended to complement unit root tests, such as the Dickey–Fuller tests. By testing both the unit root hypothesis and the stationarity hypothesis, we can distinguish series that appear to be stationary, series that appear to have a unit root, and series for which the data (or the tests) are not sufficiently informative to be sure whether they are stationary or integrated.

The main technical innovation of this paper is the allowance made for error autocorrelation. Correspondingly, the main practical difficulty in performing the tests is the estimation of the long-run variance. Our autocorrelation correction is similar to the Phillips–Perron corrections for unit root tests. In the unit root literature, the main alternatives to Phillips–Perron corrections are augmentation, instrumental variables estimation, and GLS based on either parametric (e.g., ARMA) or nonparametric models. These alternatives are worth investigating in the context of stationarity testing. Saikkonen and Luukkonen (1990) allow for autocorrelation by fitting an ARMA error structure, for example. A comparison of different methods of allowing for autocorrelation would appear to be an important topic for future research.

Appendix: Derivation of the LM statistic

Eq. (4) of the main text is eq. (1.1) of Nabeya and Tanaka (1988, p. 218) and uses their notation. For this model the LM statistic for the hypothesis $\sigma_u^2 = 0$ is given by their eq. (2.5), p. 219, as $LM = y'MD_xA_TD_xMy/y'My$. Here M is the projection matrix onto the space orthogonal to (x, Z). In our model (2), (x, Z) corresponds to intercept and time trend, so $e = My$ is the vector of residuals from a regression of y on intercept and time trend. The denominator of the statistic, $y'My$, is just the sum of squared residuals from this regression, and equals $T\hat{\sigma}_\varepsilon^2$ in the notation of the text. Apart from a factor of T, which is inessential, this is the same as the denominator of the statistic in eq. (6).

The matrix D_x equals identity when x corresponds to intercept and can therefore be ignored. The matrix A_T has (t, s)th element equal to $\min(t, s)$, so that it creates reverse partial sums. That is, the numerator of the test statistic equals

$$e'A_Te = \sum_{t=1}^{T} R_t^2, \qquad R_t = \sum_{i=t}^{T} e_i. \tag{A.1}$$

This appears to differ from the numerator of the statistic in eq. (6) of the main text, which relies on the forward partial sums S_t defined in (5). However, the two expressions are in fact equal. Because the sum of the residuals is zero, we have $R_1 = S_T = 0$, $S_t = -R_{t+1}$ $(t = 1, 2, \ldots, T-1)$, and the sum of squares of the S_t equals the sum of squares of the R_t.

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